Ad-Soyad: == CEVAP ANAHTARI == **09.01.2019**

Numara:

KODLAMA TEORİSİ I FİNAL SORULARI

1) a) Her $x, y \in \mathbb{F}_2^n$ için $d(x,y) = w(x+y)$ olduğunu gösteriniz.

b) $$\Pi: \mathbb{F}_q^n \longrightarrow \mathbb{F}_q[x]/\langle x^n - 1\rangle$$

$$a = (a_0, a_1, \ldots, a_{n-1}) \longmapsto \Pi(a) = a_0 + a_1 x + \cdots + a_{n-1}x^{n-1} + \langle x^n - 1\rangle$$

şeklinde tanımlı Π dönüşümü izomorfizma olmak üzere $\Pi(C)$, $\mathbb{F}_q[x]/\langle x^n - 1\rangle$ in bir ideali ise $\emptyset \neq C \subseteq \mathbb{F}_q^n$ devirli bir koddur. Gösteriniz.

2) $x^9 - \bar{1} \in \mathbb{F}_2[x]$ polinomunun çarpanlara ayrılışı

$$x^9 - \bar{1} = (x + \bar{1})(x^2 + x + \bar{1})(x^6 + x^3 + \bar{1})$$

olduğuna göre 9 uzunluğunda boyutu 6 olan devirli kodun, $g(x)$ üreteç polinomu ve üreteç matrisini belirleyiniz.

3) C, $r = 4$ olan $\mathbb{F}_2$ üzerindeki tanımlı Hamming kodu için

a) $n = ?$, $k = ?$, $d = ?$

b) $H = ?$

c) $(\bar{1}, \bar{1}, \bar{0}, \bar{1}, \bar{0}, \bar{1}, \bar{1}, \bar{0}, \bar{1}, \bar{1}, \bar{0}, \bar{1}, \bar{0}, \bar{0}, \bar{1})$ vektörünü dekodlayınız.

4) $\mathbb{F}_3$ üzerinde tanımlı bir lineer [5,3]-kodunun üreteç matrisi

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$$

olsun.

a) $H = ?$

b) Sendrom arama tablosunu oluşturunuz.

c) $(\bar{1}, \bar{1}, \bar{1}, \bar{1}, \bar{1}), (\bar{0}, \bar{1}, \bar{2}, \bar{1}, \bar{0})$ vektörlerini dekodlayınız

5) 7-li [8,6]-Hamming kodu için kontrol matrisini yazarak $(\bar{3}, \bar{5}, \bar{2}, \bar{3}, \bar{4}, \bar{1}, \bar{0}, \bar{6})$ ve $(\bar{1}, \bar{0}, \bar{5}, \bar{2}, \bar{1}, \bar{3}, \bar{6}, \bar{0})$ vektörlerini dekodlayınız.

BAŞARILAR

1) a) $\forall\ x, y \in \mathbb{F}_2^n$

$$d(x,y) = |\{ i : x_i \neq y_i \}|$$
$$= |\{ i : x_i - y_i \neq \bar{0} \}$$
$$= |\{ i : x_i - y_i = \bar{1} \}|$$
$$= |\{ i : x_i + y_i = \bar{1} \}| = w(x+y)$$

b) $\Pi(C)$, $\mathbb{F}_q[x] / \langle x^n - 1 \rangle$ in bir ideali olsun.

- C, lineer mi ?
- C, devirli küme mi ?

— • $\emptyset \neq C \subseteq \mathbb{F}_q^n$

• $\forall a, b \in C$, $\forall \alpha, \beta \in \mathbb{F}_q$ için $\alpha a + \beta b \in C$ mi?

$$a \in C \Longrightarrow \Pi(a) \in \Pi(C)$$
$$b \in C \Longrightarrow \Pi(b) \in \Pi(C)$$

$$\Pi(\alpha a + \beta b) = \Pi(\alpha a) + \Pi(\beta b)$$
$$= \alpha \Pi(a) + \beta \Pi(b) \in \Pi(C)$$

$\therefore$ C lineerdir

• $\forall a = (c_0, c_1, \dots, c_{n-1}) \in C \Longrightarrow (c_{n-1}, c_0, c_1, \dots, c_{n-2}) \in C$ mi?

$$a \in C \Longrightarrow \Pi(a) \in \Pi(C)$$
$$\Pi(a) = c_0 + c_1 x + \dots + c_{n-1} x^{n-1} + \langle x^n - 1 \rangle = \overline{a(x)}$$

$$\left.\begin{array}{l} \bar{x} \in \mathbb{F}_q[x] / \langle x^n - 1 \rangle \\ \\ \Pi(C) \subseteq \mathbb{F}_q[x] / \langle x^n - 1 \rangle \\ \text{ideal} \end{array}\right\} \Longrightarrow \bar{x} \cdot \overline{a(x)} \in \Pi(C)$$

$$\Longrightarrow (c_{n-1}, c_0, c_1, \dots, c_{n-2}) \in C$$

$\therefore$ C, devirli bir kümedir.

$\therefore$ C, devirli bir koddur.

2) $n = 9 \qquad k = 6 \qquad der(g(x)) = 9 - 6 = 3 \qquad q = 2$

$$g(x) = (x + \bar{1})(x^2 + x + \bar{1})$$
$$= x^3 + \bar{1}$$

$$G = \begin{bmatrix} 1&0&0&1&0&0&0&0&0 \\ 0&1&0&0&1&0&0&0&0 \\ 0&0&1&0&0&1&0&0&0 \\ 0&0&0&1&0&0&1&0&0 \\ 0&0&0&0&1&0&0&1&0 \\ 0&0&0&0&0&1&0&0&1 \end{bmatrix}_{6 \times 9}$$

3) a) $n = 2^r - 1 = 2^4 - 1 = 15 \qquad d = d(\text{Ham}(r, 2)) = 3$

$k = n - r = 15 - 4 = 11$

b) $1 = 1.2^0 + 0.2^1 + 0.2^2 + 0.2^3$

$2 = 0.2^0 + 1.2^1 + 0.2^2 + 0.2^3$

$3 = 1.2^0 + 1.2^1 + 0.2^2 + 0.2^3$

$\vdots \qquad \vdots \qquad \vdots$

$14 = 0.2^0 + 1.2^1 + 1.2^2 + 1.2^3$

$15 = 1.2^0 + 1.2^1 + 1.2^2 + 1 2^3$

$$H = \begin{bmatrix} 0&0&0&0&0&0&0&1&1&1&1&1&1&1&1 \\ 0&0&0&1&1&1&1&0&0&0&0&1&1&1&1 \\ 0&1&1&0&0&1&1&0&0&1&1&0&0&1&1 \\ 1&0&1&0&1&0&1&0&1&0&1&0&1&0&1 \end{bmatrix}_{4 \times 15}$$

c) $x = (\bar{1}, \bar{1}, \bar{0}, \bar{1}, \bar{0}, \bar{1}, \bar{1}, \bar{0}, \bar{1}, \bar{1}, \bar{0}, \bar{1}, \bar{0}, \bar{0}, \bar{1})$

$S(x) = (0, 1, 1, 0)$

$y = x - e = x - (\bar{0}, \bar{0}, \bar{0}, \bar{0}, \bar{0}, \bar{1}, \bar{0}, \bar{0}, \bar{0}, \bar{0}, \bar{0}, \bar{0}, \bar{0}, \bar{0}, \bar{0})$

$= (\bar{1}, \bar{1}, \bar{0}, \bar{1}, \bar{0}, \bar{0}, \bar{1}, \bar{0}, \bar{1}, \bar{1}, \bar{0}, \bar{1}, \bar{0}, \bar{0}, \bar{1})$

4) a) $H = \begin{bmatrix} 0 & 1 & 2 & 1 & 0 \\ 2 & 0 & 1 & 0 & 1 \end{bmatrix}_{2\times 5}$ $\quad n=5 \quad k=3$

$q=3$

b)

Sınıf Liderleri	Sendromlar
$(\bar{0},\bar{0},\bar{0},\bar{0},\bar{0})$	$(0,0)$
$(\bar{0},\bar{0},\bar{0},\bar{0},\bar{1})$	$(0,1)$
$(\bar{0},\bar{0},\bar{0},\bar{0},\bar{2})$	$(0,2)$
$(\bar{0},\bar{0},\bar{0},\bar{1},\bar{0})$	$(1,0)$
$(\bar{0},\bar{0},\bar{0},\bar{2},\bar{0})$	$(2,0)$
$(\bar{0},\bar{0},\bar{1},\bar{0},\bar{0})$	$(2,1)$
$(\bar{0},\bar{0},\bar{2},\bar{0},\bar{0})$	$(1,2)$
$(\bar{0},\bar{0},\bar{0},\bar{1},\bar{1})$	$(1,1)$
$(\bar{0},\bar{0},\bar{0},\bar{2},\bar{2})$	$(2,2)$

c) $x=(\bar{1},\bar{1},\bar{1},\bar{1},\bar{1})$

$S(x)=(1,1)$

$y = x - e = (\bar{1},\bar{1},\bar{1},\bar{1},\bar{1}) - (\bar{0},\bar{0},\bar{0},\bar{1},\bar{1})$

$= (\bar{1},\bar{1},\bar{1},\bar{0},\bar{0})$

$x_1 = (\bar{0},\bar{1},\bar{2},\bar{1},\bar{0})$

$S(x_1) = (0,2)$

$y_1 = x_1 - e = (\bar{0},\bar{1},\bar{2},\bar{1},\bar{0}) - (\bar{0},\bar{0},\bar{0},\bar{0},\bar{2})$

$= (\bar{0},\bar{1},\bar{2},\bar{1},\bar{1})$

5) $q = 7 \qquad n = 8 \qquad k = 6$

$$n = \frac{q^r - 1}{q - 1} \Longrightarrow 8 = \frac{7^r - 1}{6} \Longrightarrow r = 2$$

$$1 = 1 \cdot 7^0 + 0 \cdot 7^1$$

$$\left.\begin{array}{l} 2 = 2 \cdot 7^0 + 0 \cdot 7^1 \\ \vdots \\ 6 = 6 \cdot 7^0 + 0 \cdot 7^1 \end{array}\right\} \text{lineer bağımlı}$$

$$7 = 0 \cdot 7^0 + 1 \cdot 7^1$$

$$8 = 1 \cdot 7^0 + 1 \cdot 7^1 \qquad H = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

$$9 = 2 \cdot 7^0 + 1 \cdot 7^1$$

$$10 = 3 \cdot 7^0 + 1 \cdot 7^1$$

$$11 = 4 \cdot 7^0 + 1 \cdot 7^1$$

$$12 = 5 \cdot 7^0 + 1 \cdot 7^1$$

$$13 = 6 \cdot 7^0 + 1 \cdot 7^1$$

$$x = (\bar{5}, \bar{5}, \bar{2}, \bar{3}, \bar{4}, \bar{1}, \bar{0}, \bar{6})$$

$$S(x) = (0,0) \Longrightarrow x = y$$

$$x_1 = (\bar{1}, \bar{0}, \bar{5}, \bar{2}, \bar{1}, \bar{3}, \bar{6}, \bar{0})$$

$$S(x_1) = (3,6) = 3(1,2)$$

$$y_1 = x_1 - e = (\bar{1}, \bar{0}, \bar{5}, \bar{2}, \bar{1}, \bar{3}, \bar{6}, \bar{0}) - (\bar{0}, \bar{0}, \bar{0}, \bar{3}, \bar{0}, \bar{0}, \bar{0}, \bar{0})$$

$$= (\bar{1}, \bar{0}, \bar{5}, \bar{6}, \bar{1}, \bar{3}, \bar{6}, \bar{0})$$